

11/27.

A fibration in $\text{Sicat}_{\text{Bergner}}$ $f: \mathcal{C} \rightarrow \mathcal{D}$

- (i) $\mathcal{C}(x, y) \rightarrow \mathcal{D}(fx, fy)$ is a Kan fibration.
- (ii) $ho\mathcal{C} \rightarrow ho\mathcal{D}$ is an isofibration.

In the case $\mathcal{D} = \underline{0}$ $\begin{cases} ob = \{*\} \\ Hom(*, *)_{\mathcal{D}} = \Delta^0 \end{cases}$

then (i) \Rightarrow (ii)

$ho(\underline{0}) = [\underline{0}] = \{ * \xrightarrow{id} * \}$

$\Rightarrow ho(\mathcal{C})$ is an isofib. $\downarrow [\underline{0}] = ho(\mathcal{D})$

iff everything in $ho\mathcal{C}$ is a iso.

iff $ho\mathcal{C}$ is a gpd

iff \mathcal{C} is a ω -gpd

iff \mathcal{C} is a Kan cpx - \mathcal{M} .

X Kan enriched $\Rightarrow X$ is a fibration in $\text{Sicat}_{\mathbb{Z}}$

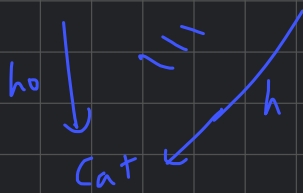
WTS (i) \checkmark & (ii) \checkmark

(\mathcal{K} is a Kan cpx.)

($\mathcal{K} \downarrow_{\Delta^0}$ is a Kan fibration)

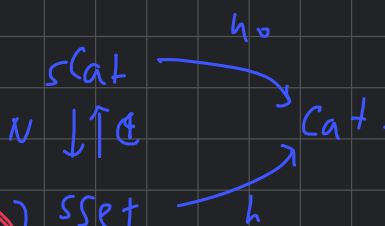
$\text{Sicat} \xrightarrow{N^{hc}} \text{Sset}$

$hoN^{hc} = ho$



$\mathcal{C} \xrightarrow{N^{hc}} \mathcal{N}\mathcal{C}$
 $ho\mathcal{C} \xrightarrow{hoN^{hc}} ho\mathcal{N}\mathcal{C}$
 $ob = ob\mathcal{C}$
 $Hom(x, y) = \pi_0 \mathcal{C}(x, y)$

$0 = ob\mathcal{C}$
 $1 = Hom_{\text{set}}(\Delta^1, \mathcal{N}\mathcal{C}) = Hom_{\text{Sicat}}(\mathcal{C}[\Delta^1], \mathcal{C})$



$h\mathcal{N}\mathcal{C} \xrightarrow{hN^{hc}} h\mathcal{N}\mathcal{C}$
 $ob = N^{hc}\mathcal{C}_0 = ob\mathcal{C}$
 $Hom_{hN^{hc}}(x, y) = N^{hc}\mathcal{C}_1 / \mathcal{N}_2$

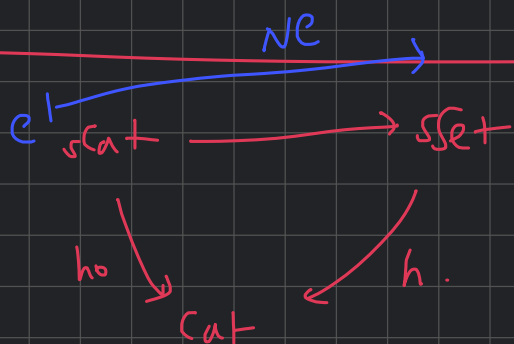
When \mathcal{C} is Kan enriched, there's mapping spaces. $\text{Map}_e(x,y)$

& $h\mathcal{C}$ has Hom sets given by connected components. $\text{Hom}_{h\mathcal{C}}(x,y) = \pi_0 \text{Map}_e(x,y)$.

$$\text{Hom}_{\text{sSet}}(\mathcal{C}[1], \mathcal{C}) \cong \pi_0 \text{Map}_e(x,y)$$

$\text{sCat} \quad \text{sSet}$
 $\mathcal{C}[n] \rightsquigarrow \Delta^n$
 $\mathcal{C}[1] \rightsquigarrow \Delta^1$

$$\mathcal{C}[1] =$$



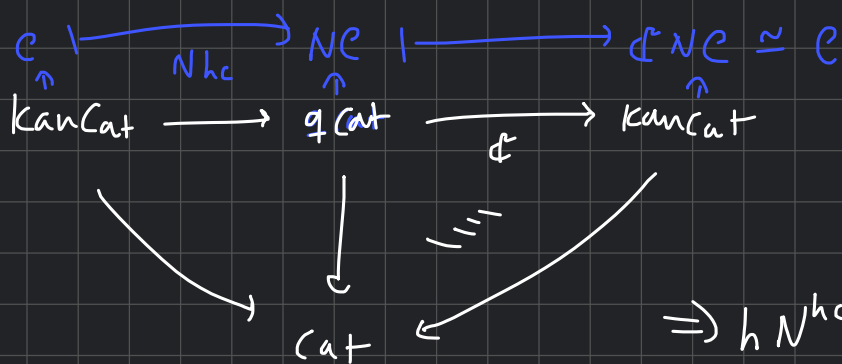
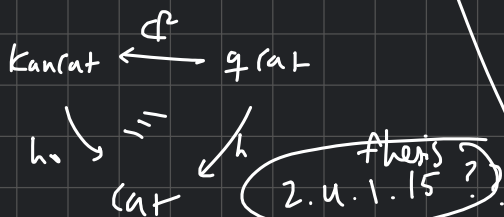
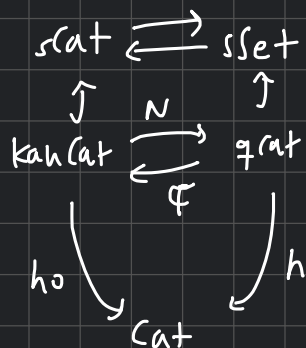
Given an \mathcal{A} -cat \mathcal{C} , $x,y \in \text{ob}(\mathcal{C})$.

$$\text{Hom}_{h\mathcal{C}}(x,y) \cong \pi_0 \text{Map}_{NE}(x,y) \cong \pi_0 \text{Hom}_{\mathcal{C}}(x,y)$$

$$\text{Map}_{NE}(x,y) = \text{Hom}_{\mathcal{C}}(x,y) \quad ?$$

Mapping space of a quasi-cat.
 $\text{Map}_{NE}(x,y) \rightarrow \text{Fun}(\Delta^1, \mathcal{C})$
 $\downarrow \quad \quad \downarrow (s,t)$
 $\Delta^0 \xrightarrow{(x,y)} \mathcal{C} \times \mathcal{C}$
 a p.b. of ssets.

$$\mathcal{C} \in \text{sCat} \implies \text{Hom}_{\mathcal{C}}(x,y) \in \text{sSet}$$



$$\implies h N^{hc} \mathcal{C} \cong \text{ho}^{s\text{Cat}}(\mathcal{C} N \mathcal{C}) \cong \text{ho}^{s\text{Cat}}(\mathcal{C})$$

from Cisinski thesis.

since $\mathcal{C} N \mathcal{C} \cong \mathcal{C}$
 Quillen adjunction.
 $(\mathcal{C} \dashv N)$

§. 1. Homotopy theories, model cat, ss-cat.

A homotopy theory is (C, W) . [We can describe homotopy theories as a homotopy theory.]

two ways to "model them"

1.1. model cats.

1.2. ss-cats.

1.2.1. kan-cat

scat.

1.2.2

g-cats

ssets.

$RelCat \longrightarrow scat \longrightarrow gcat$

$C \underset{Q}{\sim} D \longmapsto LHe \underset{DK}{\sim} LH \longmapsto NLHe \underset{w.e.g.}{\sim} NLH$

by $scat \underset{Reyn}{\sim} sset \underset{Joyal}{\sim} ?$

[HTT, 2.2.5.1].